KA-TP-12-2001

Gauge invariant decomposition of 1-loop multiparticle scattering amplitudes

Alessandro Vicini[1]
Institut für Theoretische Physik, Universität Karlsruhe, D-76128 Karlsruhe (Germany)

Abstract: A simple algorithm is presented to decompose any 1-loop amplitude for scattering processes of the class 2 fermions→ 4 fermions into a fixed number of gauge-invariant form factors. The structure of the amplitude is simpler than in the conventional approaches and its numerical evaluation is made faster. The algorithm can be efficiently applied also to amplitudes with several thousands of Feynman diagrams.
PACS: 12.15.L, 02.70

The complete calculation of the electroweak radiative corrections to the class of processes $e^+e^- \to 4f$ is still missing, for several reasons of theoretical but also especially technical origin. At 1-loop the probability amplitude is given by the sum of several thousands of Feynman diagrams, that we try to organize in a sensible way. The algebraic programs which should perform the simplifications find severe obstacles, because they have to deal with huge expressions: it is indeed very difficult to look for simplification patterns, taking the amplitude as a whole. We propose the opposite approach: having a physically motivated structure in mind, we can apply it systematically to every single Feynman diagram. The simplification of a small expression is very efficient and the bookkeeping of the various contributions follows from the beginning a precise pattern. The physical picture we are thinking of is the following: the interaction of elementary fermionic neutral and charged currents, which are factorized in the amplitude, is described by a rank-3 Lorentz tensor, which can be evaluated either at tree- or at 1-loop level and can be decomposed in a gauge-invariant way.

The paper is organized in the following way. In section 1 the present approaches and their efficiency in dealing with the scattering amplitudes are briefly described. In section 2 the decomposition proposed in this paper is formulated, proving the gauge invariance of the coefficients. In section 3 the algorithm to reduce any 1-loop Feynman diagram into the proposed form is described, and in section 4 we make some final remarks.

1 Present approaches

The number of Feynman diagrams which contribute to the probability amplitude of a process of the class $e^+e^- \to 4f$ is very large. In the following we consider only processes with massless external fermions; in table 1 we list the number of diagrams which are part of the virtual corrections to some representative processes, omitting the tadpoles contributions. One first comment is that it does not make sense to consider, pictorially, Feynman diagrams as building blocks of the calculations. As the well known example of on-shell W-pair production shows, individual Feynman diagrams contain unitarity-violating terms, which cancel in the sum at the level of the amplitude; the interference between different diagrams yields most of the physical contributions.

process	diagrams	spinor products	form factors	form factor + eq. of motion
$e^+e^- \to \mu^-\overline{\nu}_\mu u\overline{d}$	1907	2069	64	16
$e^+e^- \to e^+e^- d\overline{d}$	8522	4056	128	34
$e^+e^- \to e^+e^-e^+e^-$	21444	9157	384	104

Table 1: Some representative processes of the class $e^+e^- \to 4f$ and the number of Feynman diagrams due to their 1-loop virtual corrections, excluding tadpoles contributions, in the limit of massless external particles.

[1] vicini@particle.uni-karlsruhe.de

Since most of the processes of the class $e^+e^- \to 4f$ are mediated by the exchange of a pair of massive vector bosons, a convenient approach [1] is to expand the amplitude about the poles of resonant vector bosons propagators: one obtains double-resonant, single-resonant and non-resonant terms, whose coefficients are gauge invariant, on mathematical grounds. This expansion describes the amplitude, considering the structure of the resonances of the intermediate states. On the other hand, a lot of information about the kinematics of the process is contained in the external fermion lines. The spinor products which appear in a complete 1-loop calculation are very much involved and can not be easily simplified. We would like to concentrate our attention on this point.

In processes of the class $2f \to 2f$ it is convenient, from the computational point of view, to square the amplitude and to evaluate the traces of the spinor products, to sum over the spin polarizations. Due to the very large number of diagrams and to the length of the resulting traces, the same strategy is not so efficient when we consider processes of the class $2f \to 4f$. In this case it is common practice to adopt the helicity-amplitude technique, which consists in the projection of the amplitude on the helicity states, and then in its numerical evaluation. To square the amplitude is then just a trivial product of complex numbers and the unpolarized cross-section is obtained summing incoherently over all helicity states. Despite the last sum, the number of operations in the latter approach is considerably smaller than the one with the conventional trace technique.

Still, the helicity-state approach is not satisfactory as soon as we consider a full 1-loop amplitude. Each diagram contains the spinors and the fermion lines of the initial and final-state fermions, which interact by the exchange of gauge bosons. To simplify the amplitude, all the internal Lorentz indices are contracted (e.g. [3]). As a result, each Feynman diagram has a factor proportional to some spinor products, of the form $S = [\overline{v}\Gamma_1 u]\ [\overline{u}\Gamma_2 v]\ [\overline{u}\Gamma_3 v]$. The Γ_i are products of up to 3 Dirac matrices, contracted with the momenta flowing in the internal fermion propagators or with those present in some interaction vertices; u and v denote the particle and anti-particle spinors. The large number of Feynman diagrams yields a correspondingly large number of distinct factors S, each of them describing the specific kinematics of its parent diagram. Even after a heavy use of the Dirac algebra and of the Fierz identities, it is difficult to reduce the number of distinct factors S much below the number of Feynman diagrams of the process. The Fierz identities express a redundancy typical of these massless spinor products. Our aim is to cast all expressions into a form which is unique with respect to a given basis and which is suitable for a simple semi-classical interpretation.

In tree-level calculations the problem of the large number of spinor products has been circumvented by the observation that there are basic functions [2], which describe the spinorial part of diagrams with 0 or 1 trilinear gauge boson interaction vertices. In this way only two basic functions have to be used, where different diagrams are expressed by appropriate permutations of the arguments of these functions. The same approach could be followed at 1-loop as well, but presents some disadvantages. In fact the number of basic spinorial functions is larger, depending on the number of internal propagators in the loop: there are 4 basic functions with box integrals, 2 with pentagon integrals and 1 with hexagon integrals; self-energy and vertex corrections respect the structure of the Born diagrams. Unfortunately the use of these basic functions does not help at all in solving the questions of gauge and unitarity cancellations: only some combinations of diagrams are well behaved in the high energy limit, but the use of basic functions does not provide an algorithm to find the proper combination of diagrams.

2 Gauge invariant decomposition of the amplitude

The decomposition proposed in this paper is based on the observation that each external fermion line should be decomposed in the basis of the Dirac γ matrices. Since we consider external massless fermions, only the elements γ^μ and $\gamma^\mu\gamma_5$ are relevant. Any product of 3, 5, 7, ... γ matrices can be reduced to a combination of those two elements of the basis of the algebra. For convenience we trade these two elements with the right- and left-handed combinations $\gamma^\mu\omega_\pm$, where $\omega_\pm = (1 \pm \gamma_5)/2$. For example, we can write $\overline{u}\ \not{a}\not{b}\gamma^\mu\not{c}\not{d}\ v = c_1(a,b,c,d)\ [\overline{u}\ \gamma^\mu\omega_+\ v]\ +\ c_2(a,b,c,d)\ [\overline{u}\ \gamma^\mu\omega_-\ v]$ where c_1 and c_2 are two scalar expressions. Since we consider massless external fermions, the chiral projections $\omega_\pm\ v$ coincide with the helicity projections of the spinors.

In this section we present the general structure of the 1-loop amplitude of processes of the class $e^+e^- \to 4f$. We present first some specific example, discuss the gauge invariance of the coefficients of the decomposition, and then formulate a general rule.

Example 1: $e^+(p_1)\ e^-(p_2) \to \mu^-(p_3)\ \overline{\nu}_\mu(p_4)\ u(p_5)\ \overline{d}(p_6)$
We claim that each helicity amplitude $\mathcal{M}^{(\lambda)}$, parametrized by the index λ can be written as:

$$
\begin{aligned}
\mathcal{M}^{(\lambda)} &= J_\alpha^{12(\lambda)}\ J_\beta^{34(\lambda)}\ J_\gamma^{56(\lambda)}\ \sum_{i=1}^{64} s_i\ w_i^{\alpha\beta\gamma} \qquad (1)\\
J_\alpha^{12(\lambda)} &= \overline{v}_\lambda(p_1)\gamma_\alpha u_\lambda(p_2), \quad J_\beta^{34(\lambda)} = \overline{u}_\lambda(p_3)\gamma_\beta v_\lambda(p_4), \quad J_\gamma^{56(\lambda)} = \overline{u}_\lambda(p_5)\gamma_\gamma v_\lambda(p_6)
\end{aligned}
$$

The 64 tensors $w_i^{\alpha\beta\gamma}$ are given by all combinations of 4 independent vectors q_j^μ, namely $q_l^\alpha\ q_m^\beta\ q_n^\gamma\ \ (l,m,n = 1,\ldots,4)$. The coefficients s_i are scalar coefficients, functions of the external momenta and of the masses of the internal particles. Since we assume the vectors q_j^μ to be independent of each other, they form a complete basis of tensors of rank-3. We will discuss in the next section how to choose the momenta q_j^μ. The helicity amplitude $\mathcal{M}^{(\lambda)}$ is a physical quantity and is therefore gauge invariant. Neither the external currents J nor the tensors w_i depend on the choice of the gauge parameter. Since we decompose a gauge-invariant quantity in a basis of linearly independent tensors, each coefficient of this decomposition has to be separately gauge invariant.
Example 2: $e^+(p_1)\ e^-(p_2) \to e^+(p_3)\ e^-(p_4)\ d(p_5)\ \overline{d}(p_6)$
In a way analogous to the previous example, we write:

$$
\begin{aligned}
\mathcal{M}^{(\lambda)} &= J_\alpha^{12(\lambda)}\ J_\beta^{34(\lambda)}\ J_\gamma^{56(\lambda)}\ \sum_{i=1}^{64} s_i w_i^{\alpha\beta\gamma} \ +\ J_\alpha^{13(\lambda)}\ J_\beta^{24(\lambda)}\ J_\gamma^{56(\lambda)}\ \sum_{i=1}^{64} t_i w_i^{\alpha\beta\gamma}\\
&= J_\gamma^{56(\lambda)} \sum_{i=1}^{64} w_i^{\alpha\beta\gamma}\left(s_i J_\alpha^{12(\lambda)}\ J_\beta^{34(\lambda)}\ +\ t_i J_\alpha^{13(\lambda)}\ J_\beta^{24(\lambda)}\right) \qquad (2)\\
J_\alpha^{12(\lambda)} &= \overline{v}_\lambda(p_1)\gamma_\alpha u_\lambda(p_2), \quad J_\beta^{34(\lambda)} = \overline{u}_\lambda(p_3)\gamma_\beta v_\lambda(p_4), \quad J_\gamma^{56(\lambda)} = \overline{u}_\lambda(p_5)\gamma_\gamma v_\lambda(p_6)\\
J_\alpha^{13(\lambda)} &= \overline{v}_\lambda(p_1)\gamma_\alpha v_\lambda(p_3), \quad J_\beta^{24(\lambda)} = \overline{u}_\lambda(p_4)\gamma_\beta u_\lambda(p_2)
\end{aligned}
$$

As in the previous example, the coefficient of each tensor w_i has to be separately gauge invariant. With only few exceptions in the phase space, the tensors $J_\alpha^{12(\lambda)}\ J_\beta^{34(\lambda)}$ and $J_\alpha^{13(\lambda)}\ J_\beta^{24(\lambda)}$ are independent of each other. Therefore the two coefficients s_i and t_i have to be separately gauge-invariant, in order to make a combination gauge-invariant as well. The coefficients s_i and t_i are separately gauge-invariant, even when the two tensors $J_\alpha^{12(\lambda)}\ J_\beta^{34(\lambda)}$ and $J_\alpha^{13(\lambda)}\ J_\beta^{24(\lambda)}$ are linearly dependent, because the functional dependence which guarantees the gauge cancellation remains the same throughout the whole phase-space.
Example 3: $e^+(p_1)\ e^-(p_2) \to e^+(p_3)\ e^-(p_4)\ e^+(p_5)\ e^-(p_6)$

$$
\begin{aligned}
\mathcal{M}^{(\lambda)} = \sum_{i=1}^{64} w_i^{\alpha\beta\gamma}\Big(& s_i^{(1)} J_\alpha^{12(\lambda)}\ J_\beta^{34(\lambda)}\ J_\gamma^{56(\lambda)}\ +\ s_i^{(2)} J_\alpha^{12(\lambda)}\ J_\beta^{36(\lambda)}\ J_\gamma^{45(\lambda)}\ +\ t_i^{(1)} J_\alpha^{13(\lambda)}\ J_\beta^{24(\lambda)}\ J_\gamma^{56(\lambda)}\ +\\
& t_i^{(2)} J_\alpha^{13(\lambda)}\ J_\beta^{26(\lambda)}\ J_\gamma^{45(\lambda)}\ +\ u_i^{(1)} J_\alpha^{15(\lambda)}\ J_\beta^{24(\lambda)}\ J_\gamma^{36(\lambda)}\ +\ u_i^{(2)} J_\alpha^{15(\lambda)}\ J_\beta^{26(\lambda)}\ J_\gamma^{34(\lambda)}\Big) \qquad (3)
\end{aligned}
$$

The structure of this formula derives from the presence of identical particles in the initial and in the final state. Each expression in round brackets has to be separately gauge-invariant. As in the previous example, we observe that for generic points in the phase space the 6 different tensors $J_\alpha J_\beta J_\gamma$ are independent of each other; therefore the dependence on the gauge parameter has to cancel separately in each coefficient $s,\ t,\ u$.
General rule
Each Feynman diagram can be reduced, with the algorithm proposed in the next section, to one current product, which has initial and final state fermions connected in the same way as the external fermion lines of the diagram. Each diagram contributes to the 64 coefficients of the tensors $w_i^{\alpha\beta\gamma}$ of its specific current product. Since the current products are in general independent of each other, the proof of the gauge-invariance of the scalar coefficients holds for each of them separately.

3 Reduction algorithm

The algorithm to reduce any external fermion line to a current J_α works in 4 dimensions and might have some troubles in dimensional regularization, because of the γ_5 problem. We regularize soft-infrared and collinear divergences by means of photon and fermion masses. Tree level, virtual 1-loop diagrams with box, pentagon and hexagon loop integrals are ultraviolet finite and can be manipulated in 4 dimensions. Self-energy and vertex corrections can be considered after renormalization: namely our approach can be applied once the limit $n \to 4$ has been taken, n being the number of space-time dimensions.

1. Since we consider massless external fermions, each external fermion line contains the product of an odd number of γ matrices. We reduce these products in terms of one single γ matrix, by repeated use of the Chrisolm identity:

$$\gamma^\alpha\gamma^\beta\gamma^\gamma \;=\; \left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\gamma} + i\gamma_5\varepsilon^{\alpha\beta\gamma\delta}\right)\gamma_\delta \;\equiv\; \left(S^{\alpha\beta\gamma\delta} + i\gamma_5\varepsilon^{\alpha\beta\gamma\delta}\right)\gamma_\delta \;\equiv\; X^{\alpha\beta\gamma\delta}\gamma_\delta \tag{4}$$

The case with 5 matrices is obtained in 2 steps and gives:

$$\begin{aligned}
&\gamma^\alpha\,\gamma^\beta\,\gamma^\gamma\,\gamma^\delta\,\gamma^\zeta \;= \\
&\Big[\left(\varepsilon^{\gamma\delta\zeta\mu}g^{\alpha\beta} - \varepsilon^{\beta\delta\zeta\mu}g^{\alpha\gamma} + \varepsilon^{\alpha\delta\zeta\mu}g^{\beta\gamma} + \varepsilon^{\alpha\beta\gamma\mu}g^{\delta\zeta} - \varepsilon^{\alpha\beta\gamma\zeta}g^{\delta\mu} + \varepsilon^{\alpha\beta\gamma\delta}g^{\zeta\mu}\right) i\;\gamma_5 + \\
&g^{\alpha\mu}\,g^{\beta\zeta}\,g^{\gamma\delta} + g^{\alpha\zeta}\,g^{\beta\mu}\,g^{\gamma\delta} + g^{\alpha\mu}\,g^{\beta\delta}\,g^{\gamma\zeta} - g^{\alpha\delta}\,g^{\beta\mu}\,g^{\gamma\zeta} - g^{\alpha\zeta}\,g^{\beta\delta}\,g^{\gamma\mu} + \\
&g^{\alpha\delta}\,g^{\beta\zeta}\,g^{\gamma\mu} + g^{\alpha\mu}\,g^{\beta\gamma}\,g^{\delta\zeta} - g^{\alpha\gamma}\,g^{\beta\mu}\,g^{\delta\zeta} + g^{\alpha\beta}\,g^{\gamma\mu}\,g^{\delta\zeta} - g^{\alpha\zeta}\,g^{\beta\gamma}\,g^{\delta\mu} + \\
&g^{\alpha\gamma}\,g^{\beta\zeta}\,g^{\delta\mu} - g^{\alpha\beta}\,g^{\gamma\zeta}\,g^{\delta\mu} + g^{\alpha\delta}\,g^{\beta\gamma}\,g^{\zeta\mu} - g^{\alpha\gamma}\,g^{\beta\delta}\,g^{\zeta\mu} + g^{\alpha\beta}\,g^{\gamma\delta}\,g^{\zeta\mu}\Big]\gamma_\mu
\end{aligned} \tag{5}$$

We observe that for processes of the class $e^+e^- \to 4f$, lines with a product of 7 γ matrices appear only in the form $\gamma_\mu\gamma^\alpha\,\gamma^\beta\,\gamma^\gamma\,\gamma^\delta\,\gamma^\zeta\gamma^\mu \;=\; -2\,\gamma^\zeta\,\gamma^\delta\,\gamma^\gamma\,\gamma^\beta\,\gamma^\alpha$. We apply substitutions eq. (4) and eq. (5) to the 3 fermion lines of a Feynman diagram belonging to our class of processes and observe that after the contraction of all Lorentz indices the result has the structure $J_\alpha J_\beta J_\gamma R^{\alpha\beta\gamma}$; the tensor $R^{\alpha\beta\gamma}$ still contains metric and Levi-Civita tensors and the external momenta.

2. We choose 4 vectors q_j^μ independent of each other and write

$$g^{\mu\nu} = \frac{1}{a\overline{a}}\sum_{i,j}(a_i\cdot a_j)\,q_i^\mu q_j^\nu \tag{6}$$

$$\varepsilon^{\alpha rst} = \frac{1}{a}\left\{\left(a_{1\lambda}\cdot\varepsilon^{\lambda rst}\right)q_1^\alpha \;+\; \left(a_{2\lambda}\cdot\varepsilon^{\lambda rst}\right)q_2^\alpha \;+\; \left(a_{3\lambda}\cdot\varepsilon^{\lambda rst}\right)q_3^\alpha \;+\; \left(a_{4\lambda}\cdot\varepsilon^{\lambda rst}\right)q_4^\alpha\right\} \tag{7}$$

$$a = q_{1\alpha}q_{2\beta}q_{3\gamma}q_{4\delta}\;\varepsilon^{\alpha\beta\gamma\delta} \;\equiv\; \varepsilon^{q_1q_2q_3q_4},\quad \overline{a} = \varepsilon_{q_1q_2q_3q_4},$$
$$a_1^\mu = \varepsilon^{\mu q_2q_3q_4},\quad a_2^\mu = \varepsilon^{q_1\mu q_3q_4},\quad a_3^\mu = \varepsilon^{q_1q_2\mu q_4},\quad a_4^\mu = \varepsilon^{q_1q_2q_3\mu}$$

The product $a\overline{a}$ is the (Gram-) determinant of the matrix $q_i\cdot q_j$, while the products $a_i\cdot a_j$ yield the minors of the latter. We notice the the contraction of two Levi-Civita tensors yields a combination of scalar products. Equation (7) is the dual of the Schouten identity and expresses an axial vector in terms of polar vectors. The quantity a is pseudo-scalar with respect to parity transformations. Using the substitution rules eq. (6) and eq. (7) we are able to express all the metric and Levi-Civita tensors which appear in $R^{\alpha\beta\gamma}$ in terms of the vectors q_j^α. The tensor $R^{\alpha\beta\gamma}$ contains now only combinations of the vectors q_j and of the external momenta.

3. In order the decompositions eq. (6) and eq. (7) to hold, we need 4 independent Lorentz-vectors, so that the pseudo-scalars $a, \overline{a}$ do not vanish. It is convenient to identify the q_j^μ with some of the external momenta, to avoid cumbersome algebraic expressions and to exploit the equation of motion of the external spinors. It is important to check that the chosen momenta remain independent in the whole phase-space, to avoid artificial numerical instabilities. In Example 1 the choice $q_1 = p_3,\;\; q_2 = p_4,\;\; q_3 = p_5,\;\; q_4 = p_6$ is valid everywhere except when both $W's$ are at rest. In the latter case it is possible to have configurations in which the decay fermions of one W are parallel to those of the second W. On the other hand it is important to observe that such configurations form a negligible fraction of the phase-space and that the amplitude has a much simpler expression, thanks to the reduced number of final state momenta. Also these configurations can therefore be separately, easily, evaluated.

In the problem we have 6 external momenta p_i. By means of energy-momentum conservation we are left with 5 momenta, for instance we write $p_1 = -p_2 + p_3 + p_4 + p_5 + p_6$. Observing that a 4-dimensional space-time is completely spanned by 4 independent momenta, we decompose one momentum in terms of the other four, in our example $p_2 = c_{23}p_3 + c_{24}p_4 + c_{25}p_5 + c_{26}p_6$. With this choice for the vectors q, the tensor $R^{\alpha\beta\gamma}$ contains only factors $p_l^\alpha p_m^\beta p_n^\gamma$, $(l, m, n = 3, \ldots, 6)$, which form (with the *caveat* mentioned above) a complete basis of rank-3 Lorentz tensors, but not metric or Levi-Civita tensors. With this choice for the q's, the number of terms of the decomposition is reduced to 16 in example 1, to 34 in example 2 and to 104 in example 3.
A more solid, but less practical, solution consists in choosing $q_1 = (1, 0, 0, 0)$, $q_2 = (0, 1, 0, 0)$, $q_3 = (0, 0, 1, 0)$, $q_4 = (0, 0, 0, 1)$. In this way, equations eq. (6) and eq. (7) always hold, but we are forced to decompose all the external momenta along this special basis.

4 Discussion

1) The proposed decomposition organizes the probability amplitude in a fixed number of terms: depending on the final state this number ranges from 64 to 384 and can be further reduced by means of the equations of motion. This number has to be compared with the the one of Feynman diagrams of the corresponding reactions, or with the number of spinor products which result, after simplification, in the conventional helicity amplitude approach; they are both more than one order of magnitude larger (cf. table 1). This number is fixed *a priori* and depends only on the number of identical particles in initial and final states. In particular, it is not related at all with the number of Feynman diagrams of the reaction.

2) Each coefficient in the decomposition can be evaluated at the lowest order, or including the 1-loop corrections. The latter can be expanded, if wished, according to the pole expansion.

3) The coefficients of the decomposition are by construction gauge invariant. This property enforces naturally, in the symbolic manipulation, some of the expected gauge cancellations, because the terms which have to cancel are collected together by the decomposition algorithm. As a consequence, the numerical evaluation of the coefficients is more stable and accurate.

4) The size of the amplitude remains big, as compared with the usual approach, but its structure is much simpler. The factorization of the currents reflects a semi-classical behaviour of the amplitude, on top of which the radiative corrections act. This factorization is not trivial, because of the presence of fermion lines with up to 7 γ matrices. By semi-classical behaviour we mean that the current products give a simple and intuitive pictorial description of the fermion and flavour flows. Even if it is possible to reduce further the products $J_\alpha J_\beta J_\gamma w^{\alpha\beta\gamma}$ in terms of simpler spinor products of the form $\overline{u}(p_i)v(p_j)$, the proposed decomposition seems preferable, precisely because it does not mix particles with different flavour. In our decomposition the scalar coefficients express the weight of a given configuration of physical currents.

5) The algorithm is very efficient, because it works on one Feynman diagram at a time, with a simple set of substitution rules. Collecting together the contributions to a given scalar coefficient does not create problems from the computational point of view, even in the case of several thousands of diagrams.

6) The amount of CPU-time needed to evaluate the amplitude is reduced with respect to the normal helicity amplitude approach. In fact the number of spinor products which appear in the final expression is much smaller than in the previous case; the algebraic complexity induced by the rules eq. (4), eq. (5), eq. (6) and eq. (7) does not cost a relevant increase of the CPU-time, because the number of scalar products between the external momenta remains exactly the same. In the case of $e^+e^- \to \mu^-\overline{\nu}_\mu u\overline{d}$ this saving can be quantified to be approximately 15%. This number is not negligible, if one considers that the evaluation of one point of total cross-section requires about 1 week, on a 600 MHz Digital Sigma Station.

7) The introduction of the full set of electroweak 1-loop corrections into event generators seems to be not viable, as long as the CPU-time needed to evaluate the squared matrix elements in 1 point of the phase-space is of the order of 1 second. The only realistic alternative is to perform a careful evaluation of the differential cross-section and then to fit the latter using some appropriate function. Although the scalar coefficients s, t, u are not observable

quantities, they give a weight to the different kinematical configurations of the external currents. This relation to physical quantities, supported by their gauge-invariance, indicates that they are good candidates to become the building blocks of the calculation, instead of the Feynman diagrams. Using, for instance, a function organized as in the pole expansion, it should be possible to fit very accurately each scalar coefficient.

5 Conclusions

In this paper we have presented a simple algorithm to organize any 1-loop scattering amplitude of processes of the class $e^+e^- \to 4f$ in a physically motivated way: we decompose it in gauge invariant form factors which are suitable for analytical checks or for numerical evaluation and subsequent fits. A package implementing this algorithm has been written and tested [4].

As a first application the 1-loop amplitude for the process $e^+e^- \to \mu^-\overline{\nu}_\mu u\overline{d}$ will be presented in a forthcoming publication, its detailed description being well beyond the scope of this paper.

6 Acknowledgments

We thank O. Brein, T. Hahn, W. Hollik and D. Stöckinger for carefully reading the manuscript and useful discussions. This work was supported by the Deutsche Forschungsgemeinschaft (Forschergruppe "Quantenfeldtheorie, Computeralgebra und Monte Carlo Simulation").

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